

SOLVING THE WORD PROBLEM OF THE O'HARE MONOID FAR, FAR AWAY FROM SWEET HOME CHICAGO

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Abstract

In 1932, W. Magnus proved his classic result that each one-relator group has a (recursively) soluble word problem. With the gradual development of semigroup theory – and especially its combinatorial aspects, such as semigroup presentations – the quest for a semigroup-theoretical counterpart of the Magnus theorem became one of the most important problems in the area: is it true that all one-relator monoids have soluble word problems? A wealth of particular cases were proved by Adjan in 1966 (subsequently reproved by Lallement and Zhang, among others), and in 1987 Adjan and Oganessian reduced the problem to presentations of a particular type.

Further breakthrough happened in 2001, when Ivanov, Margolis and Meakin first proved that the one-relator monoid problem reduces to the word problem of one-relator *special inverse monoids*, with presentations of the form $\text{Inv}\langle A \mid w = 1 \rangle$. Furthermore, the word problem for such inverse monoids reduces to the membership problem for *prefix submonoids* of one-relator groups (where an inverse monoid presented as above corresponds to the prefix monoid membership problem for the group given by the same presentation, $\text{Gp}\langle A \mid w = 1 \rangle$). This is a very interesting problem on its own in the theory of one-relator groups, which has been considered and solved in numerous cases.

The O'Hare (inverse) monoid is given by the presentation

$$\text{Inv}\langle a, b, c, d \mid abcdacdadabbcdacd = 1 \rangle,$$

and it was specifically designed by Margolis and Meakin in the 80s (while waiting for a connecting flight at the O'Hare Chicago International Airport), as a geometrically bad-behaving special inverse monoid that resisted (up to now, as far as we know) attempts at its word problem, and even at one point threatened the E -unitary conjecture by Margolis, Meakin and Stephen. I am going to present the results of the joint work (still very much) in progress with Robert D. Gray (East Anglia) pertaining to the tools used in solving the word problem of the O'Hare monoid. These tools involve certain factorisations of the relator word related to the prefix submonoid and a geometrical analysis of partial Cayley graphs obtained from Stephen's procedure. Eventually, we 'project' the considered prefix monoid to the positive submonoid of a one-relator group given by a strictly positive word, and argue that it has a decidable membership problem. We finish off the talk by proposing several related problems.

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