"JUST THE MATHS"

SLIDES NUMBER

19.1

PROBABILITY 1
(Definitions and rules)

by

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UNIT 19.1 - PROBABILITY 1

DEFINITONS AND RULES

19.1.1 INTRODUCTION

Suppose 30 high-strength bolts became mixed with 25 ordinary bolts by mistake, all of the bolts being identical in appearance.

How sure can we be that, in choosing a bolt, it will be a high-strength one ?

Phrases like "quite sure" or "fairly sure" are useless, mathematically.

Hence, we <u>define</u> a way of measuring the certainty.

In 55 simultaneous choices, 30 will be of high strength and 25 will be ordinary.

We say that, in one choice, there is a $\frac{30}{55}$ chance of success.

That is, approximately, a 0.55 chance of success.

Just over half the choices will most likely give a high-strength bolt.

Such predictions can be used, for example, to
estimate the cost of mistakes on a production line.

DEFINITION 1

The various occurrences which are possible in a statistical problem are called **"events"**.

If we are interested in one particular event, it is termed **"successful"** when it occurs and **"unsuccessful"** when it does not.

ILLUSTRATION

If, in a collection of 100 bolts, there are 30 high-strength, 25 ordinary and 45 low-strength, we can make 100 **"trials"**.

In each trial, one of three events will occur (high, ordinary or low strength).

DEFINITION 2

If, in n possible trials, a successful event occurs s times, then the number $\frac{s}{n}$ is called the **"probability of success in a single trial"**.

It is also known as the **"relative frequency of success"**.

ILLUSTRATIONS

1. From a bag containing 7 black balls and 4 white balls, the probability of drawing a white ball is $\frac{4}{11}$.
2. In tossing a perfectly balanced coin, the probability of obtaining a head is $\frac{1}{2}$.
3. In throwing a die, the probability of getting a six is $\frac{1}{6}$.
4. If 50 chocolates are identical in appearance, but consist of 15 soft-centres and 35 hard-centres, the probability of choosing a soft-centre is $\frac{15}{50} = 0.3$

19.1.2 APPLICATION OF PROBABILITY TO GAMES OF CHANCE

If a competitor in a game of chance has a probability, p, of winning, and the prize money is £m, then £mp is considered to be a fair price for entry to the game.

The quantity mp is known as the **"expectation"** of the competitor.

19.1.3 EMPIRICAL PROBABILITY

So far, all the problems discussed on probability have been **"descriptive"**; that is, we know all the possible events, the number of successes and the number of failures

In other problems, called **"inference"** problems, it is necessary either

(a) to take **"samples"** in order to infer facts about a total **"population"**;

for example, a public census or an investigation of moon-rock.

or

(b) to rely on past experience;

for example past records of heart deaths, road accidents, component failure.

If the probability of success, used in a problem, has been inferred by samples or previous experience, it is called **"empirical probability"**.

However, once the probability has been calculated, the calculations are carried out in the same way as for descriptive problems.

19.1.4 TYPES OF EVENT

DEFINITION 3

If two or more events are such that not more than one of them can occur in a single trial, they are called **"mutually exclusive"**.

ILLUSTRATION

Drawing an Ace or drawing a King from a pack of cards are mutually exclusive events; but drawing an Ace and drawing a Spade are not mutually exclusive events.

DEFINITION 4

If two or more events are such that the probability of any one of them occurring is not affected by the occurrence of another, they are called **"independent"** events.

ILLUSTRATION

From a pack of 52 cards (i.e. Jokers removed), the event of drawing and immediately replacing a red card will have a probability of $\frac{26}{52} = 0.5$; and the probability of this occurring a second time will be exactly the same. They are independent events.

However, two successive events of drawing a red card **without** replacing it are **not** independent. If the first card drawn is red, the probability that the second is red will be $\frac{25}{51}$; but, if the first card drawn is black, the probability that the second is red will be $\frac{26}{51}$.

19.1.5 RULES OF PROBABILITY

1. If $p_1, p_2, p_3, ..., p_r$ are the separate probabilities of r mutually exclusive events, then the probability that some **one** of the r events will occur is

$$p_1 + p_2 + p_3 + ... + p_r.$$

ILLUSTRATION

Suppose a bag contains 100 balls of which 1 is red, 2 are blue and 3 are black.

The probability of choosing any one of these three colours will be

$$0.06 = 0.01 + 0.02 + 0.03$$

However, the probability of drawing a spade or an ace from a pack of 52 cards will not be $\frac{13}{52} + \frac{4}{52} = \frac{17}{52}$ but $\frac{16}{52}$ since there are just 16 cards which are either a spade or an ace.

2. If $p_1, p_2, p_3, ..., p_r$ are the separate probabilities of r independent events, then the probability that **all** will occur in a single trial is

$$p_1.p_2.p_3......p_r.$$

ILLUSTRATION

Suppose there are three bags, each containing white, red and blue balls.

Suppose also that the probabilities of drawing a white ball from the first bag, a red ball from the second bag and a blue ball from the third bag are, respectively, p_1, p_2 and p_3.

The probability of making these three choices in succession is $p_1.p_2.p_3$ because they are independent events.

However, if three cards are drawn, without replacing, from a pack of 52 cards, the probability of drawing a 3, followed by an ace, followed by a red card will not be $\frac{4}{52}.\frac{4}{52}.\frac{26}{52}$.

19.1.6 CONDITIONAL PROBABILITIES

For **dependent** events, the multiplication rule requires a knowledge of the **new** probabilities of successive events in the trial, after the previous ones have been dealt with. These are called **"conditional probabilities"**.

EXAMPLE

From a box, containing 6 white balls and 4 black balls, 3 balls are drawn at random without replacing them. What is the probability that there will be 2 white and 1 black ?

Solution

The cases to consider, together with their probabilities are as follows:

(a) White, White, Black

Probability $= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{120}{720} = \frac{1}{6}$.

(b) Black, White, White

Probability $= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{120}{720} = \frac{1}{6}$.

(c) White, Black, White

Probability $= \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{120}{720} = \frac{1}{6}$.

The probability of any one of these three outcomes is therefore

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}.$$