"JUST THE MATHS"

SLIDES NUMBER

17.4

NUMERICAL MATHEMATICS 4
(Further Gaussian elimination)

by

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UNIT 17.4
NUMERICAL MATHEMATICS 4
FURTHER GAUSSIAN ELIMINATION

The **elementary** method of Gaussian Elimination for simultaneous linear equations was discussed in Unit 9.4.

We introduce, here, a more **general** method, suitable for use with sets of equations having **decimal** coefficients.

17.4.1 GAUSSIAN ELIMINATION BY "PARTIAL PIVOTING" WITH A CHECK COLUMN

First, we consider an example in which the coefficients are **integers**.

EXAMPLE

Solve the simultaneous linear equations

$$\begin{aligned} 2x + y + z &= 3, \\ x - 2y - z &= 2, \\ 3x - y + z &= 8. \end{aligned}$$

Solution

We may set out the solution, in the form of a **table** indicating each of the **"pivot elements"** in a box:

	x	y	z	constant	Σ
	$\boxed{2}$	1	1	3	7
$\frac{1}{2}$	1	-2	-1	2	0
$\frac{3}{2}$	3	-1	1	8	11
		$\boxed{\frac{-5}{2}}$	$\frac{-3}{2}$	$\frac{1}{2}$	$\frac{-7}{2}$
1		$\frac{-5}{2}$	$\frac{-1}{2}$	$\frac{7}{2}$	$\frac{1}{2}$
			1	3	4

INSTRUCTIONS

(i) Divide the coefficients of x in lines 2 and 3 by the coefficient of x in line 1 and write the respective results at the side of lines 2 and 3; (that is, $\frac{1}{2}$ and $\frac{3}{2}$ in this case).

(ii) Eliminate x by subtracting $\frac{1}{2}$ times line 1 from line 2 and $\frac{3}{2}$ times line 1 from line 3.

(iii) Repeat the process starting with lines 4 and 5.

(iv) line 6 implies that $z = 3$ and by substitution back into earlier lines, we obtain the values $y = -2$ and $x = 1$.

OBSERVATIONS

Difficulties can arise for pivot elements which are very small compared with the other quantities in the same column; the errors involved in dividing by small numbers are likely to be large.

A better choice of pivot element would be the one with the **largest** numerical value in its column.

In the next example, the working will be carried out using fractional quantities though, in practice,
decimals would normally be used instead.

EXAMPLE

Solve the simultaneous linear equations

$$\begin{aligned} x - y + 2z &= 5, \\ 2x + y - z &= 1, \\ x + 3y - z &= 4. \end{aligned}$$

Solution

	x	y	z	constant	Σ
$\frac{1}{2}$	1	-1	2	5	7
	$\boxed{2}$	1	-1	1	3
$\frac{1}{2}$	1	3	-1	4	7

On eliminating x, we obtain the new table:

	y	z	constant	Σ
$\frac{-3}{5}$	$\frac{-3}{2}$	$\frac{5}{2}$	$\frac{9}{2}$	$\frac{11}{2}$
	$\boxed{\frac{5}{2}}$	$\frac{-1}{2}$	$\frac{7}{2}$	$\frac{11}{2}$

Eliminating y takes us to the final table as follows:

z	constant	Σ
$\frac{11}{5}$	$\frac{33}{5}$	$\frac{44}{5}$

We conclude that
$11z = 33$ and, hence, $\boxed{z = 3}$.

Substituting into the second table (either line will do), we have
$5y - 3 = 7$ and, hence, $\boxed{y = 2}$.

Substituting into the original table (any line will do), we have
$x - 2 + 6 = 5$ so that $\boxed{x = 1}$.

Notes:

(i) In questions which involve decimal quantities stated to n decimal places, the calculations should be carried out to $n+2$ decimal places to allow for rounding up.

(ii) A final check on accuracy is obtained by adding the original three equations together and verifying that the solution obtained also satisfies this further equation.

In the recent example, this would be

$$4x + 3y = 10,$$

and is satisfied by $x = 1$, $y = 2$.

(iii) It is not essential to set out the solution in the form of separate tables (at each step) with their own headings. A continuation of the first table is acceptable.