“JUST THE MATHS”

SLIDES NUMBER

17.2

NUMERICAL MATHEMATICS 2
(Approximate integration (A))

by

A.J.Hobson

UNIT 17.2 - NUMERICAL MATHEMATICS 2

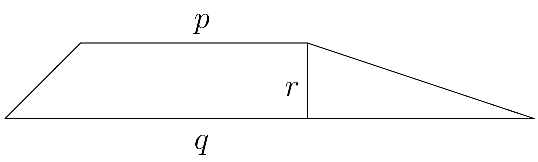
APPROXIMATE INTEGRATION (A)

17.2.1 THE TRAPEZOIDAL RULE

The Trapezoidal Rule is based on the formula for the area of a trapezium.

If the parallel sides of a trapezium are of length p and q, while the perpendicular distance between them is r, then the area A is given by

$$A = \frac{r(p+q)}{2}.$$



Suppose that the curve

$$y = f(x)$$

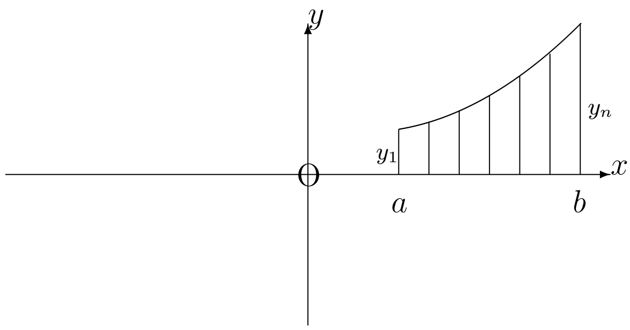
lies wholly above the x-axis between
$x = a$ and $x = b$.

The definite integral,

$$\int_a^b f(x)\,\mathrm{d}x,$$

can be regarded as the area between the curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$.

Let this area be divided into several narrow strips of equal width h by marking the values $x_1, x_2, x_3, ..., x_n$ along the x-axis (where $x_1 = a$ and $x_n = b$) and drawing in the corresponding lines of length $y_1, y_2, y_3, ..., y_n$ parallel to the y-axis



Each narrow strip of width h may be considered approximately as a trapezium whose parallel sides are of lengths y_i and y_{i+1} where $i = 1, 2, 3, ..., n-1$.

Thus, the area under the curve, and hence the value of the definite integral, approximates to

$$\frac{h}{2}[(y_1 + y_2) + (y_2 + y_3) + (y_3 + y_4) + ... + (y_{n-1} + y_n)].$$

That is,

$$\int_a^b f(x)\ \mathrm{d}x \simeq \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})].$$

Alternatively,

$$\int_a^b f(x)\ \mathrm{d}x = \frac{h}{2}[\text{First} + \text{Last} + 2 \times \ \text{The Rest}].$$

Note:
Care must be taken at the beginning to ascertain whether or not the curve $y = f(x)$ crosses the x-axis between $x = a$ and $x = b$.

If it does, then allowance must be made for the fact that areas below the x-axis are negative and should be calculated separately from those above the x-axis.

EXAMPLE

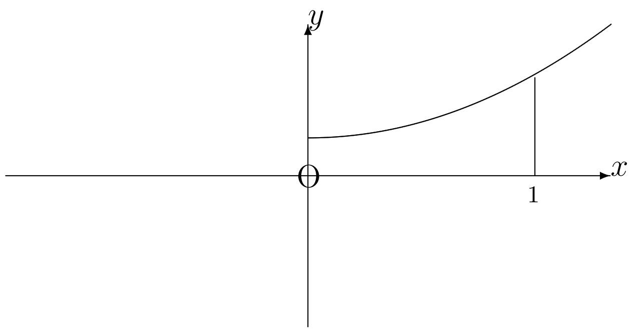
Use the trapezoidal rule with five divisions of the x-axis in order to evaluate, approximately, the definite integral:

$$\int_0^1 e^{x^2}\ \mathrm{d}x.$$

Solution

First we make up a table of values as follows:

x	0	0.2	0.4	0.6	0.8	1.0
e^{x^2}	1	1.041	1.174	1.433	1.896	2.718



Then, using $h = 0.2$,

$$\int_0^1 e^{x^2}\,\mathrm{d}x$$

$$\simeq \frac{0.2}{2}[1 + 2.718 + 2(1.041 + 1.174 + 1.433 + 1.896)]$$

$$\simeq 1.481$$