"JUST THE MATHS"

SLIDES NUMBER

16.9

Z-TRANSFORMS 2
(Inverse Z-Transforms)

by

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UNIT 16.9 - Z-TRANSFORMS 2

INVERSE Z-TRANSFORMS

16.9.1 THE USE OF PARTIAL FRACTIONS

Here, we determine a sequence, $\{u_n\}$, of numbers whose Z-Transform is a known function, $F(z)$, of z.

Such a sequence is called the **"inverse Z-Transform of $F(z)$"** and may be denoted by

$$\mathrm{Z}^{-1}[F(z)].$$

For simple difference equations, $F(z)$ is usually a rational function of z.

EXAMPLES

1. Determine the inverse Z-Transform of the function,

$$F(z) \equiv \frac{10z(z+5)}{(z-1)(z-2)(z+3)}.$$

Solution

First, we recall that

$$Z\{a^n\} = \frac{z}{z-a}.$$

Then, we write

$$F(z) \equiv z.\left[\frac{10(z+5)}{(z-1)(z-2)(z+3)}\right].$$

This gives

$$F(z) \equiv z.\left[\frac{-15}{z-1} + \frac{14}{z-2} + \frac{1}{z+3}\right]$$

or

$$F(z) \equiv \frac{z}{z+3} + 14\frac{z}{z-2} - 15\frac{z}{z-1}.$$

Hence,

$$\mathrm{Z}^{-1}[F(z)] = \left\{(-3)^n + 14(2)^n - 15\right\}.$$

2. Determine the inverse Z-Transform of the function,

$$F(z) \equiv \frac{1}{z-a}.$$

Solution

In this example, there is no factor, z, in the function, $F(z)$, and we shall see that it is necessary to make use of the first shifting theorem.

First, we may write

$$F(z) \equiv \frac{1}{z}\left[\frac{z}{z-a}\right]$$

and, since the inverse Z-Transform of the expression inside the brackets is a^n, the first shifting theorem tells us that

$$Z^{-1}[F(z)] = \begin{cases} 0 & \text{when } n = 0; \\ a^{n-1} & \text{when } n > 0. \end{cases}$$

Note:

This may now be taken as a standard result

3. Determine the inverse Z-Transform of the function,

$$F(z) \equiv \frac{4(2z+1)}{(z+1)(z-3)}.$$

Solution

Expressing $F(z)$ in partial fractions, we obtain

$$F(z) \equiv \frac{1}{z+1} + \frac{7}{(z-3)}.$$

Hence,

$$Z^{-1}[F(z)] = \begin{cases} 0 & \text{when } n = 0; \\ (-1)^{n-1} + 7.(3)^{n-1} & \text{when } n > 0. \end{cases}$$