

**“JUST THE MATHS”**

**UNIT NUMBER**

**5.10**

**GEOMETRY 10**  
**(Graphical solutions)**

by

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**5.10.1 Introduction**

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## UNIT 5.10 - GEOMETRY 10

### GRAPHICAL SOLUTIONS

#### 5.10.1 INTRODUCTION

An algebraic equation in a variable quantity,  $x$ , may be written in the general form

$$f(x) = 0,$$

where  $f(x)$  is an algebraic expression involving  $x$ ; we call it a “**function of  $x$** ” (see Unit 10.1).

In the work which follows,  $f(x)$  will usually be either a **linear** function of the form  $ax + b$ , where  $a$  and  $b$  are constants, or a **quadratic** function of the form  $ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are constants.

The solutions of the equation  $f(x) = 0$  consist of those values of  $x$  which, when substituted into the function  $f(x)$ , cause it to take the value zero.

The solutions may also be interpreted as the values of  $x$  for which the graph of the equation

$$y = f(x)$$

meets the  $x$ -axis since, at any point of this axis,  $y$  is equal to zero.

#### 5.10.2 THE GRAPHICAL SOLUTION OF LINEAR EQUATIONS

To solve the equation

$$ax + b = 0,$$

we may plot the graph of the equation  $y = ax + b$  to find the point at which it meets the  $x$ -axis.

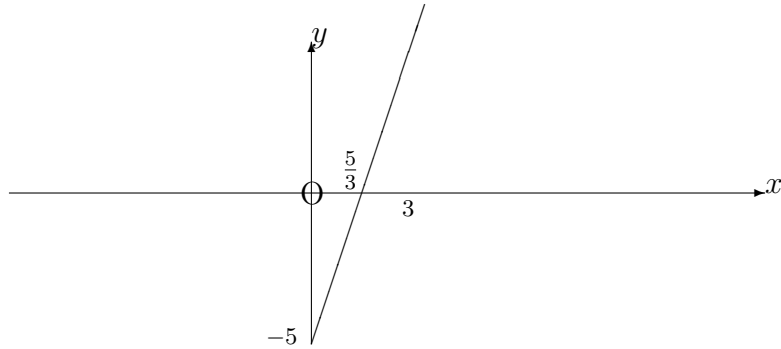
#### EXAMPLES

1. By plotting the graph of  $y = 3x - 5$  from  $x = 0$  to  $x = 3$ , solve the linear equation

$$3x - 5 = 0.$$

#### Solution

$x$	0	1	2	3
$y$	-5	-2	1	4



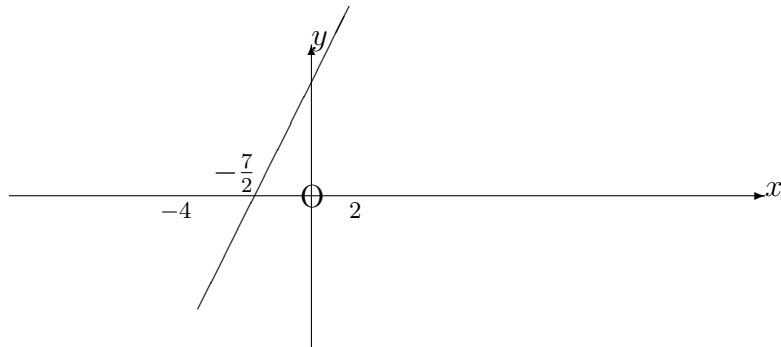
Hence  $x \simeq 1.7$

2. By plotting the graph of  $y = 2x + 7$  from  $x = -4$  to  $x = 2$ , solve the linear equation

$$2x + 7 = 0$$

**Solution**

$x$	-4	-3	-2	-1	0	1	2
$y$	-1	1	3	5	7	10	11



Hence  $x = -3.5$

### 5.10.3 THE GRAPHICAL SOLUTION OF QUADRATIC EQUATIONS

To solve the quadratic equation

$$ax^2 + bx + c = 0$$

by means of a graph, we may plot the graph of the equation  $y = ax^2 + bx + c$  and determine the points at which it crosses the  $x$ -axis.

An alternative method is to plot the graphs of the two equations  $y = ax^2 + bx$  and  $y = -c$  in order to determine their points of intersection. This method is convenient since the first graph has the advantage of passing through the origin.

#### EXAMPLE

By plotting the graph of  $y = x^2 - 4x$  from  $x = -2$  to  $x = 6$ , solve the quadratic equations

(a)

$$x^2 - 4x = 0;$$

(b)

$$x^2 - 4x + 2 = 0;$$

(c)

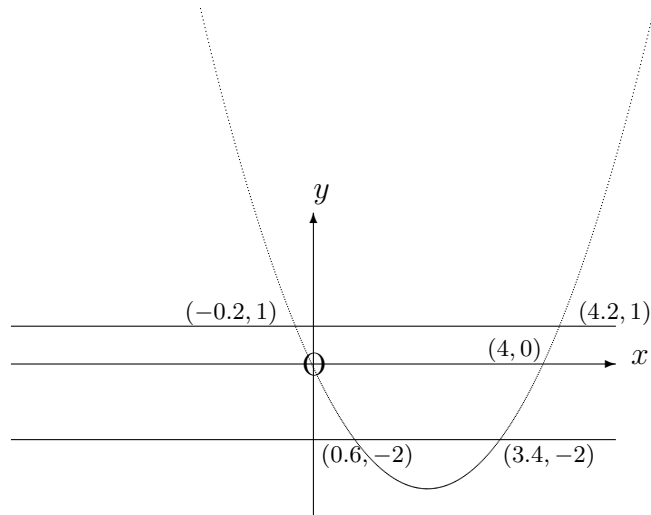
$$x^2 - 4x - 1 = 0.$$

#### Solution

A table of values for the graph of  $y = x^2 - 4x$  is

$x$	-2	-1	0	1	2	3	4	5	6
$y$	12	5	0	-3	-2	-3	0	5	12

For parts (b) and (c), we shall also need the graphs of  $y = -2$  and  $y = 1$ .



Hence, the three sets of solutions are:

(a)

$$x = 0 \text{ and } x = 4;$$

(b)

$$x \simeq 3.4 \text{ and } x \simeq 0.6;$$

(c)

$$x \simeq 4.2 \text{ and } x \simeq -0.2$$

#### 5.10.4 THE GRAPHICAL SOLUTION OF SIMULTANEOUS EQUATIONS

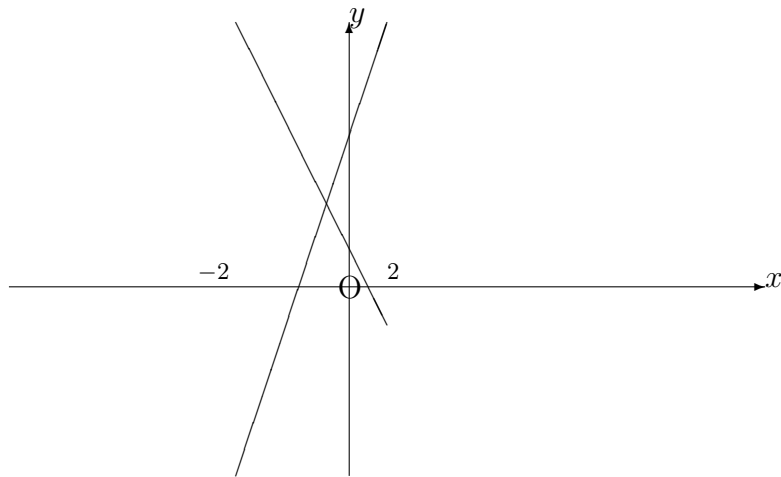
A simple extension of the ideas covered in the previous paragraphs is to solve either a pair of simultaneous linear equations or a pair of simultaneous equations consisting of one linear and one quadratic equation. More complicated cases can also be dealt with by a graphical method but we shall limit the discussion to the simpler ones.

#### EXAMPLES

1. By plotting the graphs of  $5x + y = 2$  and  $-3x + y = 6$  from  $x = -2$  to  $x = 2$ , determine the common solution of the two equations.

**Solution**

$x$	-2	-1	0	1	2
$y_1 = 2 - 5x$	12	7	2	-3	-8
$y_2 = 6 + 3x$	0	3	6	9	12



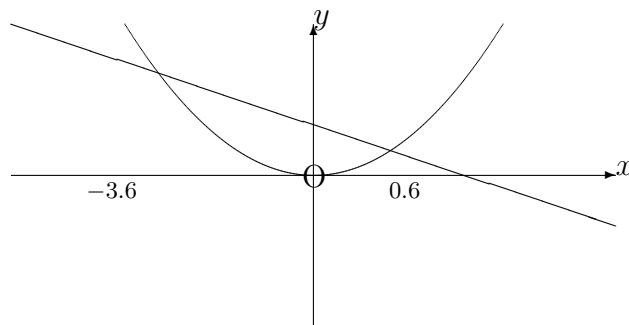
Hence,  $x = -0.5$  and  $y = 4.5$ .

2. By plotting the graphs of the equations  $y = x^2$  and  $y = 2 - 3x$  from  $x = -4$  to  $x = 2$  determine their common solutions and hence solve the quadratic equation

$$x^2 + 3x - 2 = 0.$$

**Solution**

$x$	-4	-3	-2	-1	0	1	2
$y_1 = x^2$	16	9	4	1	0	1	4
$y_2 = 2 - 3x$	14	11	8	5	2	-1	-4



Hence  $x \simeq 0.6$  and  $x \simeq -3.6$ .

### 5.10.5 EXERCISES

In these exercises, state your answers correct to one place of decimals.

1. Use a graphical method to solve the following linear equations:

(a)

$$8x - 3 = 0;$$

(b)

$$8x = 7.$$

2. Use a graphical method to solve the following quadratic equations:

(a)

$$2x^2 - x = 0;$$

(b)

$$2x^2 - x + 3 = 10;$$

(c)

$$2x^2 - x = 11.$$

3. Use a graphical method to solve the following pairs of simultaneous equations:

(a)

$$3x - y = 6 \quad \text{and} \quad x + y = 0;$$

(b)

$$x + 2y = 13 \quad \text{and} \quad 2x - 3y = 14;$$

(c)

$$y = 3x^2 \quad \text{and} \quad y = -5x + 1.$$

### 5.10.6 ANSWERS TO EXERCISES

1. (a)

$$x \simeq 0.4;$$

(b)

$$x \simeq 0.9$$

2. (a)

$$x = 0 \text{ and } x = 2;$$

(b)

$$x \simeq 2.1 \text{ and } x \simeq -1.6;$$

(c)

$$x \simeq 2.6 \text{ and } x \simeq -2.1$$

3. (a)

$$x = 1.2 \text{ and } y = -1.2;$$

(b)

$$x \simeq 9.6 \text{ and } y \simeq 1.7;$$

(c)

$$x \simeq 0.18 \text{ and } y \simeq 0.1 \text{ or } x \simeq -1.8 \text{ and } y \simeq 10.2$$