"JUST THE MATHS"

UNIT NUMBER

3.2

TRIGONOMETRY 2
(Graphs of trigonometric functions)

by

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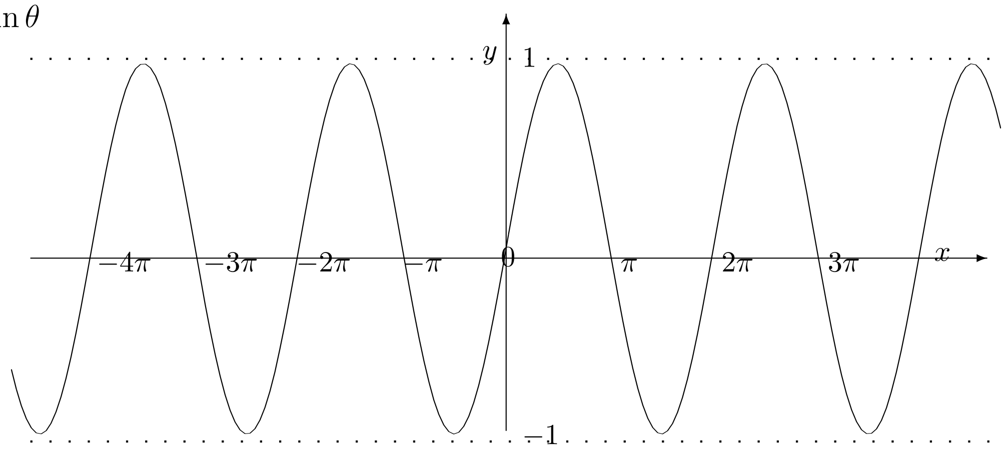
UNIT 3.2 - TRIGONOMETRY 2.
GRAPHS OF TRIGONOMETRIC FUNCTIONS

3.2.1 GRAPHS OF ELEMENTARY TRIGONOMETRIC FUNCTIONS

The following diagrams illustrate the graphs of the basic trigonometric functions sinθ, cosθ and tanθ,

1. $y = \sin\theta$



The graph illustrates that

$$\sin(\theta + 2\pi) \equiv \sin\theta$$

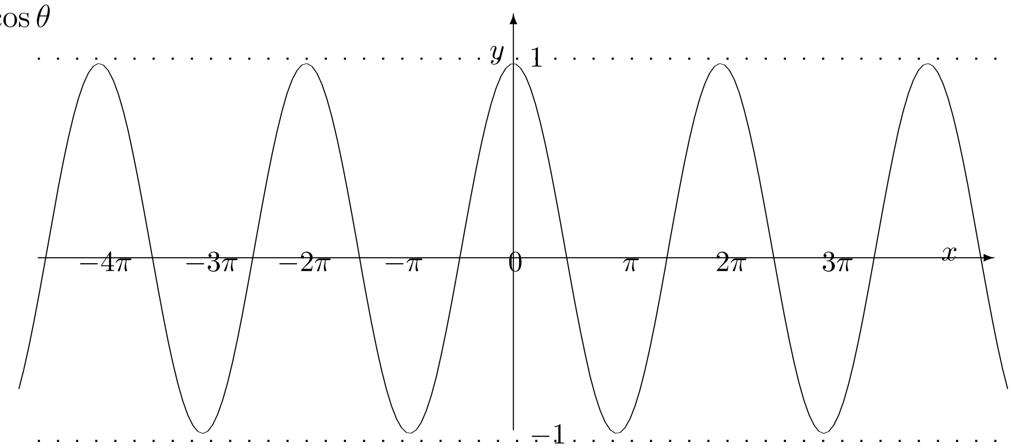
and we say that sinθ is a **"periodic function with period** 2π**"**.
Other numbers which can act as a period are $\pm 2n\pi$ where n is any integer; but 2π itself is the smallest positive period and, as such, is called the **"primitive period"** or sometimes the **"wavelength"**.

We may also observe that

$$\sin(-\theta) \equiv -\sin\theta$$

which makes sinθ what is called an **"odd function"**.

2. $y = \cos\theta$



The graph illustrates that

$$\cos(\theta + 2\pi) \equiv \cos\theta$$

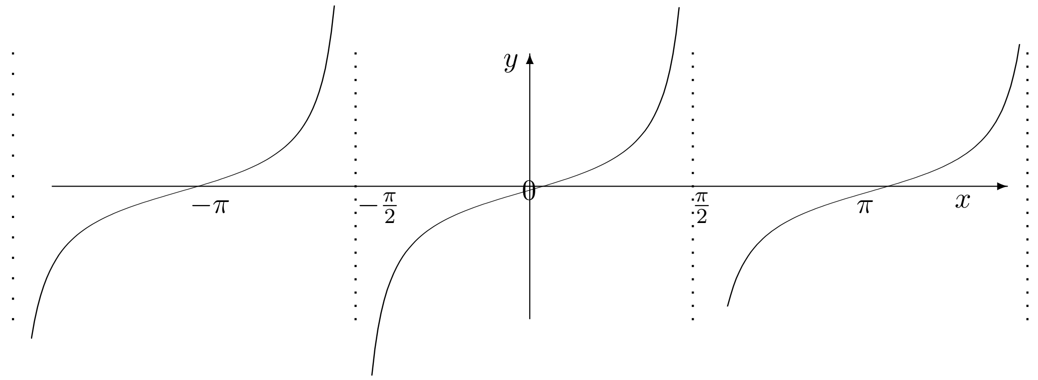
and so cosθ, like sinθ, is a periodic function with primitive period 2π

We may also observe that

$$\cos(-\theta) \equiv \cos\theta$$

which makes cosθ what is called an **"even function"**.

3. $y = \tan\theta$



This time, the graph illustrates that

$$\tan(\theta + \pi) \equiv \tan\theta$$

which implies that tanθ is a periodic function with primitive period π.

We may also observe that

$$\tan(-\theta) \equiv -\tan\theta$$

which makes tanθ an **"odd function"**.

3.2.2 GRAPHS OF MORE GENERAL TRIGONOMETRIC FUNCTIONS

In scientific work, it is possible to encounter functions of the form

$$\boxed{\text{Asin}(\omega\theta + \alpha)} \text{ and } \boxed{\text{Acos}(\omega\theta + \alpha)}$$

where ω and α are constants.

We may sketch their graphs by using the information in the previous examples 1. and 2.

EXAMPLES

1. Sketch the graph of

$$y = 5\cos(\theta - \pi).$$

Solution

The important observations to make first are that

(a) the graph will have the same shape as the basic cosine wave but will lie between $y = -5$ and $y = 5$ instead of between $y = -1$ and $y = 1$; we say that the graph has an **"amplitude"** of 5.

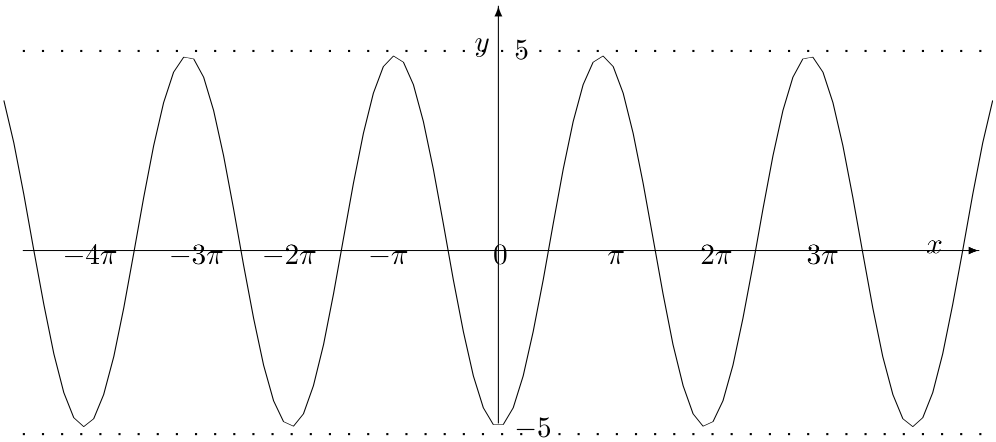
(b) the graph will cross the θ-axis at the points for which

$$\theta - \pi = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$$

that is

$$\theta = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$$

(c) The y-axis must be placed between the smallest **negative** intersection with the θ-axis and the smallest **positive** intersection with the θ - axis (in proportion to their values). In this case, the y-axis must be placed half way between $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$.



Of course, in this example, from earlier trigonometry results, we could have noticed that

$$5\cos(\theta - \pi) \equiv -5\cos\theta$$

so that graph consists of an "upsidedown" cosine wave with an amplitude of 5. However, not all examples can be solved in this way.

2. Sketch the graph of

$$y = 3\sin(2\theta + 1).$$

Solution

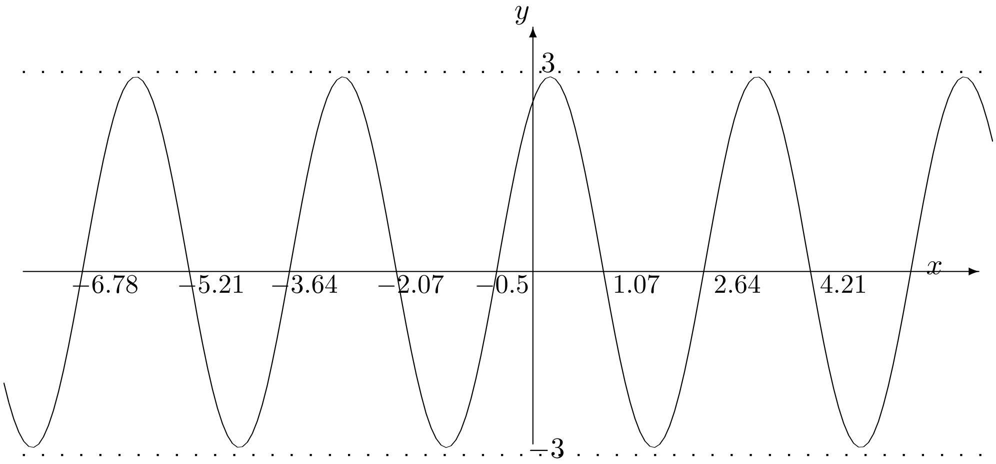
This time, the graph will have the same shape as the basic sine wave, but will have an amplitude of 3. It will cross the θ-axis at the points for which

$$2\theta + 1 = 0, \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi, ...$$

and by solving for θ in each case, we obtain

$$\theta = ... - 6.78, -5.21, -3.64, -2.07, -0.5, 1.07, 2.64, 4.21, 5.78...$$

Hence, the y-axis must be placed between $\theta = -0.5$ and $\theta = 1.07$ but at about one third of the way from $\theta = -0.5$



3.2.3 EXERCISES

1. Make a table of values of θ and y, with θ in the range from 0 to 2π in steps of $\frac{\pi}{12}$, and hence, sketch the graphs of

   (a)
   $$y = \sec\,\theta;$$

   (b)
   $$y = \text{cosec}\,\theta;$$

   (c)
   $$y = \cot\,\theta.$$

2. Sketch the graphs of the following functions:

   (a)
   $$y = 2\sin\left(\theta + \frac{\pi}{4}\right);$$

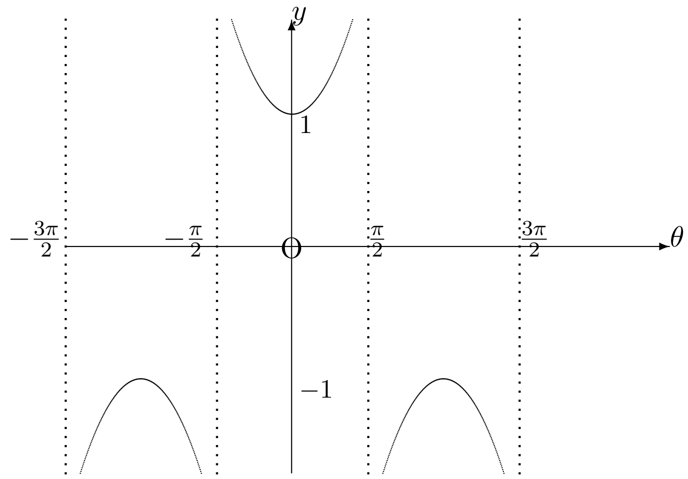
   (b)
   $$y = 2\cos(3\theta - 1).$$

   (c)
   $$y = 5\sin(7\theta + 2).$$

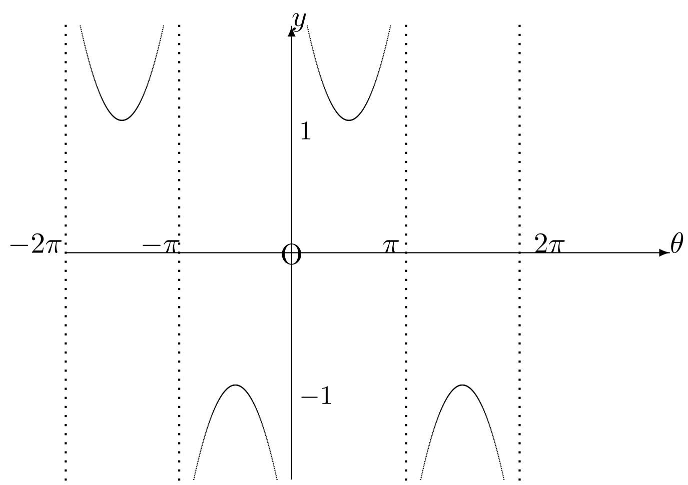
   (d)
   $$y = -\cos\left(\theta - \frac{\pi}{3}\right).$$

3.2.4 ANSWERS TO EXERCISES

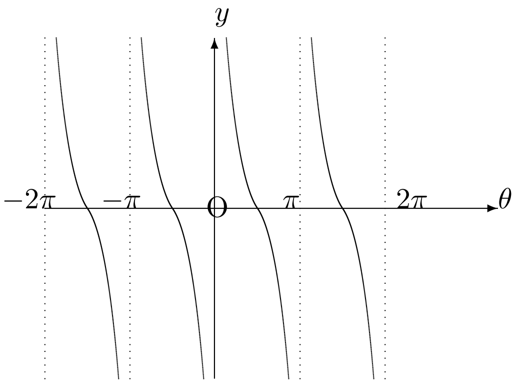
1. (a) The graph is



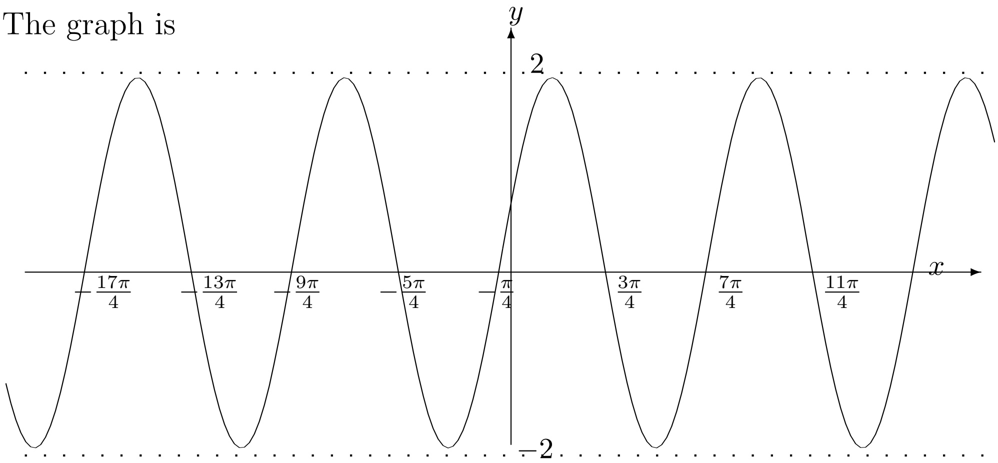
(b) The graph is



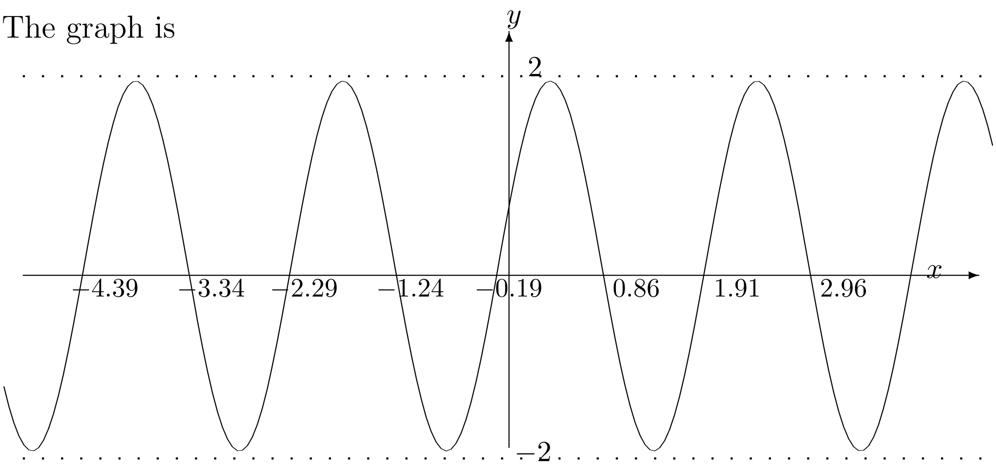
(c) The graph is



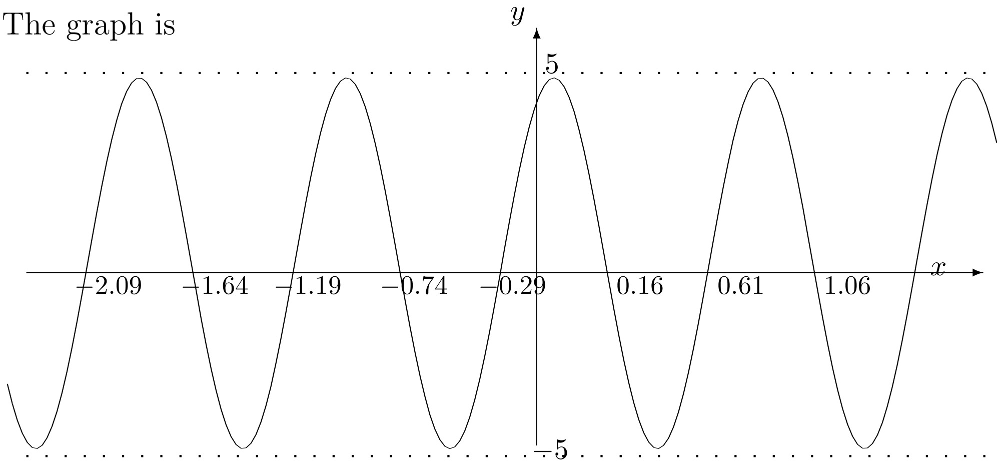
2. (a) The graph is



(b) The graph is



(c) The graph is



(d) The graph is