"JUST THE MATHS"

UNIT NUMBER

1.2

ALGEBRA 2
(Numberwork)

by

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UNIT 1.2 - - ALGEBRA 2 - NUMBERWORK

1.2.1 TYPES OF NUMBER

In this section (and elsewhere) the meaning of the following types of numerical quantity will need to be appreciated:

(a) NATURAL NUMBERS
These are the counting numbers 1, 2, 3, 4, ...

(b) INTEGERS
These are the positive and negative whole numbers and zero;
i.e. ......-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

(c) RATIONALS
These are the numbers which can be expressed as the ratio of two integers but can also be written as a terminating or recurring decimal (see also next section)

For example

$$\frac{2}{5} = 0.4$$

and

$$\frac{3}{7} = 0.428714287142871....$$

(d) IRRATIONALS
These are the numbers which cannot be expressed as either the ratio of two integers or a recurring decimal (see also next section)

Typical examples are numbers like

$$\pi \simeq 3.1415926.....$$

$$e \simeq 2.71828.....$$

$$\sqrt{2} \simeq 1.4142135....$$

$$\sqrt{5} \simeq 2.2360679....$$

The above four types of number form the system of **"real numbers"**.

1.2.2 DECIMAL NUMBERS

(a) Rounding to a specified number of decimal places
Most decimal quantities used in scientific work need to be approximated by **"rounding"** them (up or down as appropriate) to a specified number of decimal places, depending on the accuracy required.

When rounding to n decimal places, the digit in the n-th place is left as it is when the one after it is below 5; otherwise it is taken up by one digit.

EXAMPLES

1. 362.5863 = 362.586 to 3 decimal places;
   362.5863 = 362.59 to 2 decimal places;
   362.5863 = 362.6 to 1 decimal place;
   362.5863 = 363 to the nearest whole number.
2. 0.02158 = 0.0216 to 4 decimal places;
   0.02158 = 0.022 to three decimal places;
   0.02158 = 0.02 to 2 decimal places.

(b) Rounding to a specified number of significant figures
The first significant figure of a decimal quantity is the first non-zero digit from the left, whether it be before or after the decimal point.

Hence when rounding to a specified number of significant figures, we use the same principle as in **(a)**, but starting from the first significant figure, then working to the right.

EXAMPLES

1. 362.5863 = 362.59 to 5 significant figures;
   362.5863 = 362.6 to 4 significant figures;
   362.5863 = 363 to 3 significant figures;
   362.5863 = 360 to 2 significant figures;
   362.5863 = 400 to 1 significant figure.
2. 0.02158 = 0.0216 to 3 significant figures; 0.02158 = 0.022 to 2 significant figures;
   0.02158 = 0.02 to 1 significant figure.

1.2.3 THE USE OF ELECTRONIC CALCULATORS

(a) B.O.D.M.A.S.

The student will normally need to work to the instruction manual for the particular calculator being used; but care must be taken to remember the B.O.D.M.A.S. rule for priorities in calculations when pressing the appropriate buttons.

For example, in working out $7.25 + 3.75 \times 8.32$, the multiplication should be carried out first, then the addition. The answer is 38.45, not 91.52.

Similarly, in working out $6.95 \div [2.43 - 1.62]$, it is best to evaluate $2.43 - 1.62$, then generate its reciprocal with the $\frac{1}{x}$ button, then multiply by 6.95. The answer is 8.58, not 1.24

(b) Other Useful Numerical Functions

Other useful functions to become familiar with for scientific work with numbers are those indicated by labels such as $\sqrt{x}$, x^2, x^y and $x^{\frac{1}{y}}$, using, where necessary, the **"shift"** control to bring the correct function into operation.

For example:

$$\sqrt{173} \simeq 13.153;$$
$$173^2 = 29929;$$
$$23^3 = 12167;$$
$$23^{\frac{1}{3}} \simeq 2.844$$

(c) The Calculator Memory

Familiarity with the calculator's memory facility will be essential for more complicated calculations in which various parts need to be stored temporarily while the different steps are being carried out.

For example, in order to evaluate

$$(1.4)^3 - 2(1.4)^2 + 5(1.4) - 3 \simeq 2.824$$

we need to store each of the four terms in the calculation (positively or negatively) then recall their total sum at the end.

1.2.4 SCIENTIFIC NOTATION

(a) Very large numbers, especially decimal numbers are customarily written in the form

$$a \times 10^n$$

where n is a positive integer and a lies between 1 and 10.

For instance,

$$521983677.103 = 5.21983677103 \times 10^8.$$

(b) Very small decimal numbers are customarily written in the form

$$a \times 10^{-n}$$

where n is a positive integer and a lies between 1 and 10.

For instance,

$$0.00045938 = 4.5938 \times 10^{-4}.$$

Note:
An electronic calculator will allow you to enter numbers in scientific notation by using the **EXP** or **EE** buttons.

EXAMPLES

1. Key in the number 3.90816×10^{57} on a calculator.

   Press **3.90816** **EXP** **57**

   In the display there will now be 3.90816 57 or $3.90816^{\times 10^{57}}$.

2. Key in the number 1.5×10^{-27} on a calculator

   Press **1.5** **EXP** **27** +/-

   In the display there will now be 1.5 - 27 or $1.5^{\times 10^{-27}}$.

Notes:

(i) On a calculator or computer, scientific notation is also called *floating point notation.*

(ii) When performing a calculation involving decimal numbers, it is always a good idea to check that the result is reasonable and that a major arithmetical error has not been made with the calculator.

For example,

$$69.845 \times 196.574 = 6.9845 \times 10^1 \times 1.96574 \times 10^3.$$

This product can be **estimated** for reasonableness as:

$$7 \times 2 \times 1000 = 14000.$$

The answer obtained by calculator is 13729.71 to two decimal places which is 14000 when rounded to the nearest 1000, indicating that the exact result could be reasonably expected.

(iii) If a set of measurements is made with an accuracy to a given number of significant figures, then it may be shown that any calculation involving those measurements will be accurate only to one significant figure more than the least number of significant figures in any measurement.

For example, the edges of a rectangular piece of cardboard are measured as 12.5cm and 33.43cm respectively and hence the area may be evaluated as

$$12.5 \times 33.43 = 417.875\text{cm}^2.$$

Since one of the edges is measured only to three significant figures, the area result is accurate only to four significant figures and hence must be stated as 417.9cm^2.

1.2.5 PERCENTAGES

Definition
A percentage is a fraction whose denominator is 100. We use the per-cent symbol, %, to represent a percentage.

For instance, the fraction $\frac{17}{100}$ may be written 17%

EXAMPLES

1. Express $\frac{2}{5}$ as a percentage.
   Solution
   $$\frac{2}{5} = \frac{2}{5} \times \frac{20}{20} = \frac{40}{100} = 40\%$$
2. Calculate 27% of 90.
   Solution
   $$27\% \text{ of } 90 = \frac{27}{100} \times 90 = \frac{27}{10} \times 9 = 24.3$$

3. Express 30% as a decimal.

   Solution

$$30\% = \frac{30}{100} = 0.3$$

1.2.6 RATIO

Sometimes, a more convenient way of expressing the ratio of two numbers is to use a colon (:) in place of either the standard division sign (÷) or the standard notation for fractions.

For instance, the expression 7:3 could be used instead of either $7 \div 3$ or $\frac{7}{3}$. It denotes that two quantities are "in the ratio 7 to 3" which implies that the first number is seven thirds times the second number or, alternatively, the second number is three sevenths times the first number. Although more cumbersome, the ratio 7:3 could also be written $\frac{7}{3}$:1 or 1:$\frac{3}{7}$.

EXAMPLES

1. Divide 170 in the ratio 3:2

   Solution

   We may consider that 170 is made up of $3 + 2 = 5$ parts, each of value $\frac{170}{5} = 34$.

   Three of these make up a value of $3 \times 34 = 102$ and two of them make up a value of $2 \times 34 = 68$.

   Thus 170 needs to be divided into 102 and 68.

2. Divide 250 in the ratio 1:3:4

   Solution

   This time, we consider that 250 is made up of $1 + 3 + 4 = 8$ parts, each of value $\frac{250}{8} = 31.25$. Three of these make up a value of $3 \times 31.25 = 93.75$ and four of them make up a value of $4 \times 31.25 = 125$.

   Thus 250 needs to be divided into 31.25, 93.75 and 125.

1.2.7 EXERCISES

1. Write to 3 s.f.

   (a) 6962; (b) 70.406; (c) 0.0123;
   (d) 0.010991; (e) 45.607; (f) 2345.

2. Write 65.999 to

   (a) 4 s.f. (b) 3 s.f. (c) 2 s.f.
   (d) 1 s.f. (e) 2 d.p. (f) 1 d.p.

3. Compute the following in scientific notation:

   (a) $(0.003)^2 \times (0.00004) \times (0.00006) \times 5,000,000,000$;
   (b) $800 \times (0.00001)^2 \div (200,000)^4$.

4. Assuming that the following contain numbers obtained by measurement, use a calculator to determine their value and state the expected level of accuracy:

   (a)
   $$\frac{(13.261)^{0.5}(1.2)}{(5.632)^3};$$
   (b)
   $$\frac{(8.342)(-9.456)^3}{(3.25)^4}.$$

5. Calculate 23% of 124.

6. Express the following as percentages:

   (a) $\frac{9}{11}$; (b) $\frac{15}{20}$; (c) $\frac{9}{10}$; (d) $\frac{45}{50}$; (e) $\frac{75}{90}$.

7. A worker earns £400 a week, then receives a 6% increase. Calculate the new weekly wage.

8. Express the following percentages as decimals:

   (a) 50% (b) 36% (c) 75% (d) 100% (e) 12.5%

9. Divide 180 in the ratio 8:1:3

10. Divide 930 in the ratio 1:1:3

11. Divide 6 in the ratio 2:3:4

12. Divide 1200 in the ratio 1:2:3:4

13. A sum of £2600 is to be divided in the ratio $2\frac{3}{4} : 1\frac{1}{2} : 2\frac{1}{4}$. Calculate the amount of money in each part of the division.

1.2.8. ANSWERS TO EXERCISES

1. (a) 6960; (b) 70.4; (c) 0.0123;
   (d) 0.0110; (e) 45.6; (f) 2350.
2. (a) 66.00; (b) 66.0; (c) 66;
   (d) 70; (e) 66.00; (f) 66.0
3. (a) 1.08 - 04 or $1.08^{\times 10^{-4}}$ (b) 5 - 29 or $5^{\times 10^{-29}}$;
4. (a) 0.0245, accurate to three sig. figs. (b) −63.22, accurate to four sig. figs.
5. 28.52
6. (a) 81.82% (b) 75% (c) 90% (d) 90% (e) 83.33%
7. £424.
8. (a) 0.5; (b) 0.36; (c) 0.75; (d) 1; (e) 0.125
9. 120, 15, 45.
10. 186, 186, 558.
11. 1.33, 2, 2.67
12. 120, 240, 360, 480
13. £1100, £600 £900.